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SYSTEM NOISE ANALYSIS OF AN ULTRA WIDE BAND APERTURE ARRAY ELEMENT FOR LOW FREQUENCY RADIO ASTRONOMY

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ABSTRACT
One of the biggest challenges in radio astronomy for the 21st century is the *SKA telescope*, the newest radio telescope in the world to be built as a *S*quare *K*ilometre *A*rray of antennas distributed in hundreds of stations to form an interferometric image of the sky. Below 1 GHz, these stations will be composed of thousands of inexpensive elements in a dense or sparse configuration delivering data to powerful digital processing systems. Low noise temperature will be crucial to achieve the ambitious survey speeds and sensitivity aimed for by astronomers. In this paper the authors analyze an ultra-wide band antenna embedded in an infinite periodic aperture array working at frequencies from 70 to 450 MHz. The aim is to study the trade-offs in terms of noise contributions present in such array.

KEY WORDS
Phased antenna arrays, Aperture arrays, Noise temperature, Radio astronomy, Bow-Tie antennas.

1. Introduction

The so called Aperture Arrays [1] will be the core technology for the next generation Radio Telescopes. These are phased antenna arrays composed of several thousand elements which are used to directly sample the incoming wavefront. They will provide around two orders of magnitude improvement in sensitivity over existing instruments. Among these, the Square Kilometre Array (SKA) [2] will drive the radio astronomy science projects for a large portion of the century. Also, an improvement of thousand times with respect to current facilities is expected in the speed with which the sky can be imaged. This performance is required to deliver the scientific objectives of the experiment which include: mapping the distribution of neutral hydrogen gas in the universe out to very early cosmic epochs and the dawn of galaxies; studying the origin of magnetic fields and measuring the properties of all the pulsars in our galaxy to study gravitational physics. In order to develop such an instrument new technologies are required to provide very large collecting area (over one square kilometre) and antenna technology which is very wideband, and can be mass-produced at reasonable costs.

The use of phased antenna arrays in radio astronomy is not new. Such a telescope was designed and built in Cambridge and was the key technology which led to the discovery of pulsars by Hewish and Bell. It is only now with the advent of new processing technologies that an instrument on the scale of the SKA will be made possible. One particular science objective is to study the formation of the first galaxies which resulted in the cold neutral hydrogen pervading much of the universe at this time being ionized. This so-called “epoch of re-ionisation” occurs at a sufficiently early time that the signals from the 21-cm line of neutral hydrogen are “redshifted” into the frequency range from 70MHz through to several hundred MHz. One part of the SKA design therefore calls for an ultra-wide band antenna array able to operate from 70-450MHz. There are clear cost savings if this frequency range can be covered by a single antenna technology. The SKA will have full frequency coverage from 70MHz to 10GHz. Aperture array systems are also being actively developed for the 300MHz – ~1GHz range [3], while relatively small parabolic antennas will be used for the frequency range ~1GHz – 10GHz [3].

The SKA’s primary figure of merit is the system sensitivity, which is a direct measure of the ratio effective aperture to system noise (Aeff/Tsys) [3]. The system temperature is defined by the impedance mismatch between the electronics and also by the sky noise [4]. For these kinds of instruments, noise is therefore a key parameter and its trend across the frequency band gives the designer a good insight into the potential capabilities of the telescope.

In section II the authors present their approach to simulate these types of Aperture Arrays, justifying their choice of a commercial software package. In section III the results of an analysis of the system noise temperature for a unit cell of an aperture array composed of ultra-wide band bow-tie elements are presented.

2. Aperture Arrays Analysis

The system noise temperature of the SKA is a key parameter to minimize in order to achieve the desired sensitivity [3]. The system noise temperature is essentially composed of two sources; one originates in the instrument itself and the other is due to the noise collected from the sky. At low frequencies the latter is especially high, setting an upper boundary on the sensitivity. In the analysis of section III, authors have assumed a model to measure the system noise accounting for the contribution of both sources, given by

$$T_{sys} = T^a_{ant} + T^a_{rec} \tag{1}$$

Where

$$T^a_{ant} = \eta^a_{rad} \cdot T^a_A + \left(1 - \eta^a_{rad}\right) \cdot T^a_p \tag{2}$$

The noise temperature due to the antenna array is defined as T^a_{ant}. This can be subdivided into the noise created by the physical temperature of the antenna array (T^a_p) and the noise received by the antenna array from the sky and surroundings (T^a_A). These two noise temperatures are weighted by the radiation efficiency of the antenna array (η^a_{rad}). T^a_{rec} is the receiver noise temperature and it is highly dependent on the active matching between the antenna elements and the LNAs located after them and on the beam forming network. Several authors are lately referring the system noise for a receiving phased array of antennas, to the sky [5], as it is a common point in the system. Despite authors agree with their arguments, in the present paper the system noise temperature is being studied for a unit cell in an infinite periodic array and therefore it is still acceptable to assume as the reference point the output of the antenna (connection with the LNA), as usual.

2.1 Antenna noise temperature, T^a_{ant}

The antenna noise term above, T^a_A, is essentially a convolution of the antenna/array beam with the sky at a specific frequency, and can be simply understood by the following

$$T^a_A = \frac{\iint_{4\pi} \left|F(\nu,\theta,\phi)\right|^2 T_{sky}(\nu,\theta,\phi) \sin\theta d\theta d\phi}{\iint_{4\pi} \left|F(\nu,\theta,\phi)\right|^2 \sin\theta d\theta d\phi} \tag{3}$$

For an array, the radiation pattern is given by the $|F|^2$ term which can be calculated using the appropriate software (e.g. CST [6], HFSS [7], IE3D [8], MATLAB [9], etc.) The sky temperature distribution is given by the term T_{sky}.

For calculating arbitrary array performance, the model used for the sky brightness can be a spectral one. Medellin [10] describes a model which gives the sky temperature distribution as a function of frequency and angle, θ (from zenith). This model is described using radiative transfer theory and an additional term for the air mass [10].

Whilst this type of sky model is very useful, for specific array/antenna designs, a more appropriate option is to use the measured sky brightness. There are currently a number of sky surveys including the 408 MHz [11], the 1.42 GHz [12] and 2.3 GHz which are suitable for the frequency range of the SKA. The advantage of using the measured sky is that certain scenarios can be tested, such as those where bright sources pass over the side-lobes of the beam.

2.2 Receiver noise temperature, T^a_{rec}

The noise originated in the receiver chain (including the beam forming network), T^a_{rec}, is caused by the losses induced in the electronics and the impedance mismatch between the different components in the chain. At low frequencies, this noise should be much lower than the noise coming from the sky since this decreases with frequency as shown in Fig. 1 and therefore it represents a small contribution to the system temperature. In this situation, the antenna or LNA performance could be relaxed whilst keeping an acceptable T_{sys}.

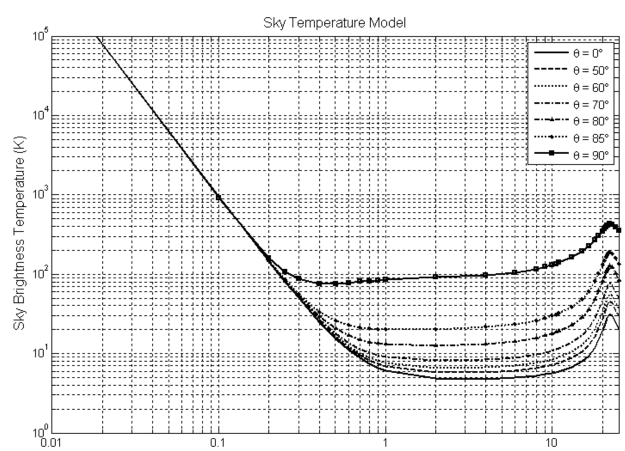


Fig. 1: Sky temperature model based on [10]

For a basic single receiver chain composed of an antenna plus an amplifier, the noise temperature, T_{rec}, maybe modeled with the following equation [13]. A detailed description of how to compute T^a_{rec} accounting for the beam forming network can be found in [14].

$$T_{rec} = \left(\left(F_{\min} + \left(\frac{R_n}{real\left(1/Z_{ant} \right)} \right) \cdot \left| 1/Z_{ant} - 1/Z_{opt} \right|^2 \right) - 1 \right) \cdot T_0 \tag{4}$$

F_{min} is the minimum noise figure of the amplifier, R_n is the noise resistance of the transistor, Z_{opt} is the optimum noise impedance, Z_{ant} is the input impedance of the antenna and T_0 is the ambient temperature, typically 290K.
Simulating phased array antennas with dimensions of thousands of wavelengths is a real challenge and a limiting factor in the design of the new generation of radio telescopes. The commercial packages available based on classical numerical methods as Method of Moments (MoM) (i.e. [8]), Finite Differences in Time Domain (FDTD) (i.e. [6]) or Finite Element Method (FEM) (i.e. [7]) are very limited on the size of the array they can simulate. For these astronomical super stations, the aforementioned commercial packages could simulate an infinite array of elements, which will be accurate enough only for the inner elements of the station, but dismissing the truncation effects [15] and assuming that the array has a regular configuration of identical elements. A new MoM based tool is being developed to reduce the simulation time for finite arrays of thousands of elements [16]. This approach provides much more accuracy than the conventional infinite array simulation approach.

The MoM efficiency can be improved by combining a method to reduce the number of unknowns such as the use of MBF's [17], with an algorithm to speed up the computations to fill the reduced matrix. An example for this is the adaptive integral method (AIM) [18][19] or the adaptive cross approximation algorithm (ACA) [20]. Also, the fast multipole method (FMM) [21] and variations are being used to speed up the matrix filling, such as the multilevel fast multipole algorithm (MLFMA) [22] which decomposes the entire volume in first-level cubical regions which will generate higher order regions when grouped.

Novel techniques such as sub-gridding [23] allow applying MoM methods to non-uniform arrays, e.g. random arrays. In this technique the smooth behaviour of the interactions between CBF's is used to interpolate them.

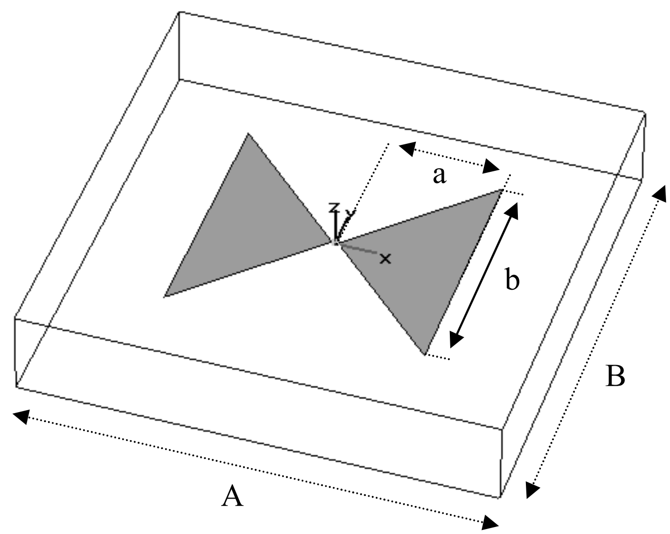
3. Numerical results

An ultra wide band low profile bow-tie element as in Fig. 2 is analyzed to check its viability as a potential element for the SKA. The element is made of PEC and it is placed a fourth of a wavelength at the highest frequency of interest (166 mm at 450 MHz) on top a ground plane to avoid the image effect through the usable frequency band [24]. The use of a ground plane will avoid undesired uncontrolled reflections in the earth and will push the radiation to be confined in the upper hemisphere pointing to the sky. The parameters of the unit cell are "a", where 2a is the size of the element along the E-plane; "b", the size of the element along the H-plane; "A", the size of the unit cell along the E-plane and "B", the size of the unit cell along the H-plane.

A regular infinite phased array of the proposed elements was simulated with the periodic boundaries approach of commercial software based on FD-TD [6] to obtain the main characteristics of the array in an infinite array environment. The impact in the performance of the inter-element spacing, *d* (Table 2), and the element size (Table 3) are analyzed.

The system noise for a single receiver (but embedded in the infinite array) as explained in section II is calculated using the spectral sky model and an idealize LNA, which will drive the response of the rest of the receiving chain. In Table 1 the parameters of this LNA are shown.

a) Unit cell



b) Feeding model

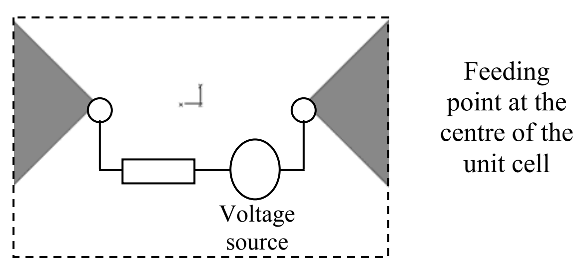


Fig. 2: Simulated unit cell of the regular array. a) Unit cell. The element type is a Bow-Tie antenna placed 166 mm on top of a ground plane. b) Feeding model.

Table 1: Idealize LNA parameters.

F_{min}	R_n	Z_{opt}	Z_{amp}
0.2	10 Ω	200 Ω	200 Ω

The model for the sky temperature distribution T_{sky} used here is based on [10]. For frequencies below 1 GHz, this model can be fitted by the following equation, where f is the frequency in GHz:

$$T(f) = 1.691 f^{-2.751} + 4.875 \quad (5)$$

The choice of a uniform sky model makes sense in the context of the analysis of a single unit cell, where the shape of the pattern itself does not strongly affect how much power is collected by the antenna from the whole sky. Furthermore, it is out of the scope of this paper analyzing specific observations of the sky, where the use of measured data would be necessary.

In Fig. 3 the system noise of a single receiver embedded in the infinite regular array is shown for different values of *d* (see Table 2). Due to the beneficial effect of mutual coupling in the antenna impedance, and assuming an ideal LNA with constant parameters, the system noise can be improved dramatically by placing elements closer in the array grid. The benefit of mutual coupling is clearly seen in Fig. 4; the difference in receiver noise at 100 MHz between an isolated element (0.28λ long at 100MHz) and the same element embedded in an infinite array with inter-element spacing of λ/2 at 150 MHz (0.33λ at 100 MHz) is 250K.

Table 2: Unit cell parameters for the first sweep – variation of the inter-element spacing

A	*B*	*a*	*b*
Sweeps between 1m and 2.140m	Sweeps between 1m and 2.140m	42.5cm	41.5cm

It is important to notice that “a” and “b” differ in 10 mm due to the feeding gap of 20mm. However, the unit cell is kept completely square and its size sweeps from 1m to 2.140m, meaning half wavelength spacing distances from 150MHz down to approximately 70MHz.

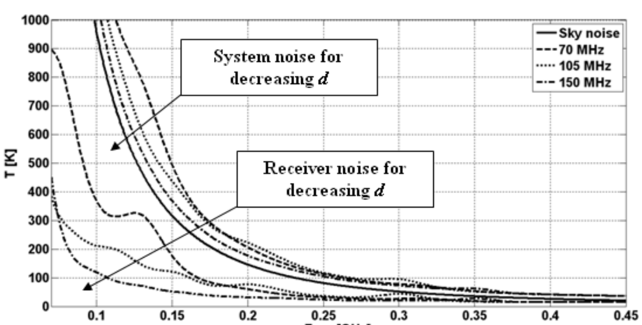


Fig. 3: Simulated unit cell system noise for a regular array configuration. In the legend is the frequency of the half wavelength spacing *d*. The solid line represents the contribution from the sky.

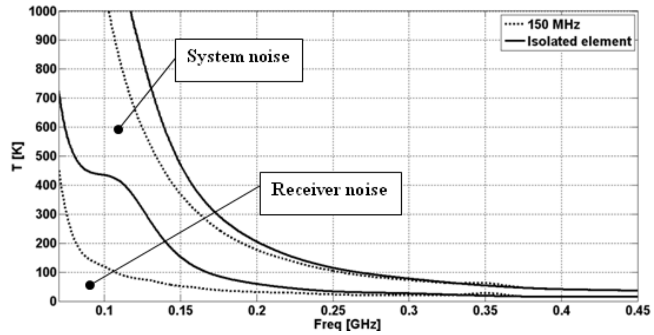


Fig. 4: Simulated unit cell system noise for a regular array configuration. In the legend is the frequency of the half wavelength spacing *d*. The solid line represents the result for an isolated antenna.

In Table 3 the parameters for the analysis of the effect of the element size for a fixed size of unit cell are shown. “*a*” and “*b*” will vary in such a way that the bow-tie element keeps a constant aspect ratio. Therefore, only “*b*” is shown in the legend of the plots.

Table 3: Unit cell parameters for the second sweep – variation of the element size.

A	*B*	*a*	*b*
1.5m	1.5m	Sweeps between 31cm and 71cm	Sweeps between 30cm and 70cm

Due to anomalies also observable in the impedance (results not shown), the system temperature rises unacceptably for a narrowband for certain antenna sizes at a frequency around 315 MHz. As shown in Fig. 5, for larger antennas, with size comparable to λ/2, the system temperature improves dramatically as one could expect.

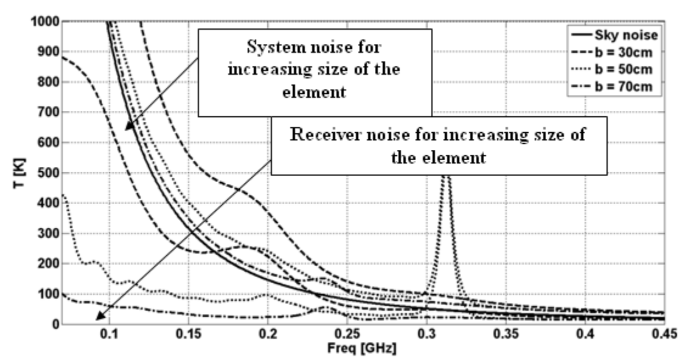


Fig. 5: Simulated unit cell system noise for a regular array configuration. In the legend is the dimension “b” of the antenna. The solid line represents the contribution from the sky.

4. Conclusion

In conclusion, this analytical study of aperture arrays unit cells refutes some well known facts about array theory, such as looking at the mutual coupling as a benefit rather than as a problem, and also gives designers some useful hints on how to plan a design of a radio astronomy station, i.e.; if interested in minimizing the system noise, larger antennas, of sizes comparable to λ/2 will likely represent better receiver temperatures, at the cost of a more expensive structure. However, a worst system temperature delivered by a smaller antenna element can be compensated by taking advantage of mutual coupling and placing antennas closer in the array grid.

Furthermore, at the very low end of the band, the sky noise dominates over the receiver noise. Also, with the exception of certain impedance narrow band anomalies, the receiver noise showed to be always lower than the sky noise, as for higher frequencies where the sky noise is low, the antenna elements are large enough to achieve reasonably good impedance matching with the LNA. In a typical case, only for frequencies above 200 MHz the sky noise is less than an order of magnitude higher than the receiver noise. As stated above, the performance of antenna and amplifier could be then relaxed at the lower end of the band, resulting in further cost saving. Therefore, it seems that the middle band, where the sky noise has decreased to values comparable to the receiver noise and this receiver noise has stabilized is the crucial frequency range to be optimized.

A bow-tie antenna element has proved to be a suitable candidate for this type of arrays, offering the necessary flexibility in a wide band of frequencies.

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