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ON THE PATHWAY TO THE SKA
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Determining neutrino properties with the SKA

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Abstract. Current measurements of cosmic large-scale structure (LSS) are able to place an ~ 1 eV upper limit on the absolute mass scale of neutrinos. A factor ~ 20 improvement in mass sensitivity, together with an insensitivity to systematic effects, is needed to reach the lowest value allowed by particle physics experiments. We consider here the prospects of determining the neutrino mass scale, the number of 'massive' neutrinos and the mass hierarchy using 'all-sky' redshift surveys with the Square Kilometre Array (SKA), with or without the addition of Cosmic Microwave Background (CMB) data from the Planck satellite. We conclude that a full understanding of the conventional neutrino sector is very likely to be achieved with a combination of LSS(SKA) and CMB(Planck) datasets.

1. Introduction

The proof that neutrinos have mass was a major breakthrough in particle physics. This proof came from the observation (e.g. Fukuda et al. 1996) that neutrinos in one weak-flavour state are able to 'oscillate into' neutrinos of a different weak-flavour state which, in 'the vacuum', is disallowed by quantum mechanics unless neutrinos have mass. The implications of this result are profound not only for particle physics, but also for cosmology. It was the first measured effect that is not included in the 'standard model' of particle physics. It also started to set constraints on the absolute mass scale of a particle species.

In this paper we investigate how measurements of the large-scale structure of HI-emitting galaxies with the Square Kilometre Array (SKA) can transform our knowledge of neutrino properties. We keep with particle physics notation by using natural units in which $c = \hbar = k = 1$. We will refer throughout to a 'fiducial' cosmology in which the normal cosmological parameters take the values: $\{\Omega_b, \Omega_c, w, h, n_s, \sigma_8\} = \{0.04, 0.26, -1, 0.72, 1.0, 0.9\}$. We define Ω_m as the fraction of critical density contributed by all matter: baryons, CDM and neutrinos. We assume that the Universe is flat. The power spectrum $P(k)$ is calculated using CAMB (Lewis et al. 2000). We reserve the use of the symbol m_ν for the absolute mass scale of neutrinos, by which we mean the value of the rest mass of the most massive neutrino, and we take N_ν to be the number of neutrino types with approximately this mass. The term Σm_i refers to summing neutrino masses over all three $(i = 1 - 3)$ mass eigenstates. For further details we refer the reader to Abdalla & Rawlings (2007).

2. Measuring neutrino properties with the SKA

2.1. A back-of-the-envelope calculation

The free streaming of neutrinos damps the power spectrum of galaxies up to scales of the size of the horizon when massive neutrinos become non-relativistic. On large scales, the magnitude of this damping is given roughly by

$$\frac{\Delta P(k)}{P(k)} \simeq -8\frac{\Omega_\nu}{\Omega_m} \simeq -8\left(\frac{m_\nu}{93.14\ \mathrm{eV}}\right)\left(\frac{N_\nu}{\Omega_m\ h^2}\right). \qquad (1)$$

Here we attempt to estimate the extent to which we would be able to measure this effect with a future large-scale-structure (LSS) survey with the SKA. We first outline a simple back-of-the-envelope calculation to estimate the cosmic volume required, and then in Sec.2.2 we undertake a more detailed calculation that takes into account the full effect that neutrinos have on both LSS and CMB datasets.

If we ignore for now the role of priors, current measurements of the galaxy power spectrum with surveys such as the 2dF Galaxy Redshift Survey (2dFGRS) and the SLOAN Digital Sky Survey (SDSS) yield upper limits around $m_\nu \sim 1$ eV (Tegmark et al. 2006). Hence, a redshift survey with ~ 400-times the cosmic volume of these surveys would have error bars on large scales a factor ~ 20-times smaller, and would therefore be able to probe to $m_\nu \sim 0.05$ eV, at the very bottom end of the range allowed by particle physics, provided the relation given by Eqn.1 holds. Such an experiment would thus be able to probe the entire neutrino sector allowed by current particle physics experiments

An 'all sky' galaxy redshift survey reaching to very high redshifts would be the definitive way to probe such signals. We consider here a $20,000\ \mathrm{deg}^2$ SKA survey reaching $z \sim 2$. Such a survey would provide the increase in cosmic volume needed to comprehensively probe neutrino properties. Although we use simulated LSS datasets from surveys with the SKA, our conclusions are valid for any spectroscopic LSS survey with a similar reach in sky area, redshift depth and galaxy number density. Further details concerning future redshift surveys with the SKA can be found in the following references: details concerning galaxy number densities in Abdalla & Rawlings (2005)

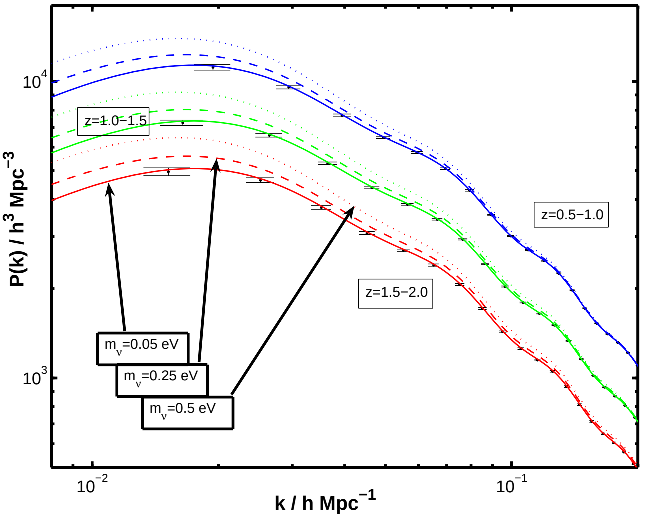

Fig. 1. Illustration of the sensitivity a future LSS (SKA) survey will need to measure and constrain the absolute neutrino mass scale m_ν. The error bars illustrate the accuracy of the SKA LSS measurement in three redshift bins, with $P(k)$ taken from a fiducial cosmological model which neglects massive neutrinos. Note that the plotted error bars should be totally uncorrelated. The solid, dashed and dotted lines correspond to the addition to the fiducial model of neutrinos of mass 0.05 eV, 0.25 eV and 0.5 eV (taking $N_\nu = 3$), all of which are still possible values given the constraints from current data sets. The different colours represent independent SKA measurements in three redshift slices: $0.5 \leq z < 1.0$ (blue, top), $1.0 \leq z < 1.5$ (green, middle) and $1.5 \leq z < 2$ (red, bottom).

and details concerning $P(k)$ measurement in Abdalla et al. (2008).

We illustrate in Fig. 1 the promise of a future LSS (SKA) survey for detecting the imprint of neutrinos on $P(k)$ in which all other cosmological parameters are fixed at fiducial values and m_ν is varied through the neutrino density parameter Ω_ν (see Eqn. 6 of Abdalla & Rawlings 2007).

The features in the power spectrum due to neutrinos could plausibly be mimicked by galaxy bias (Seljak 2000). The huge number of galaxies in an SKA-like LSS survey would allow the probing of the galaxy power spectrum with several galaxy types in several redshift shells, thus producing many independent power spectra. This will be critical for disentangling the effects of neutrinos from possible effects due to galaxy bias because all redshift shells should have the same signal arising from neutrino physics, whereas the power spectrum in each shell will have different bias properties.

We still, of course, need to test whether constraints on neutrino parameters are robust to parameter degeneracies. Any such degeneracies could make an SKA LSS experiment heavily reliant on priors unless a joint analysis is performed with other key datasets, such as those arising from future CMB experiments.

2.2. *MCMC methods probing neutrino scenarios with the SKA*

In order to probe the cosmological parameter space we use Markov Chain Monte Carlo (MCMC) methods in order to obtain a prediction of the posterior probability for a given model and a given simulated survey (assuming a fiducial model). We use the following parameters in our MCMC chains: $\{\Omega_b, \Omega_c, \Omega_\nu, h, n_s, \sigma_8, N_\nu, b_1, p_1, b_2, p_2, b_3, p_3\}$, where the last six parameters account for multiplicative bias b_j and additive shot-noise p_j terms in three ($j = 1-3$) independent redshift bins. We have conservatively assumed that the LSS data can only be trusted on relatively large comoving scales ($k < k_{\rm max} = 0.2\ h\ {\rm Mpc}^{-1}$).

We will focus on two MCMC-based studies: one which uses simulated SKA data alone and assumes an absolute mass scale for neutrinos near the top of the allowed range; and another which uses a combined simulated SKA/CMB(Planck Satellite) dataset, and assumes the lowest possible allowed value for m_ν.

2.2.1. SKA-only study if $\Sigma m_i \sim 0.25$ eV

We first choose to examine whether a survey with the SKA will be able to probe m_ν as low as ~ 0.25eV. We choose a fiducial model which has $N_\nu = 2$, and impose a uniform prior $0 < N_\nu < 3$ on the full parameter space of standard neutrino models. The fiducial bias for all three redshift bins has been conservatively set to unity (see Abdalla et al. 2008).

For an SKA survey alone, there should be a $\gtrsim 3\sigma$ detection of the absolute mass scale of the neutrino (see Fig. 6 of Abdalla & Rawlings 2007). The measurement of the power spectrum, in three separate redshift bins, is accurate enough that stringent priors are unnecessary for a detection of the neutrino signature, although problems with parameter degeneracies (see Fig. 2) remain.

2.2.2. SKA plus CMB studies

If we are assessing models which have small values of Ω_ν then the suppression of the power at small scales is very small and can be mimicked by a change in n_s in the primordial power spectrum. Hence, a LSS survey may be unable to distinguish between a signal left over from an inflationary phase of the Universe and a signal due to neutrinos.

We illustrate in Fig. 2 the Ω_ν - n_s degeneracy present in a future SKA survey on its own. This degeneracy depends on the choice of the smallest scales on which the galaxy power spectrum can be trusted to reflect the underlying dark-matter $P(k)$, i.e. $k_{\rm max}$, however Planck will be able to probe high k values by measuring high l values. As we can see from Fig. 2, Planck will measure n_s

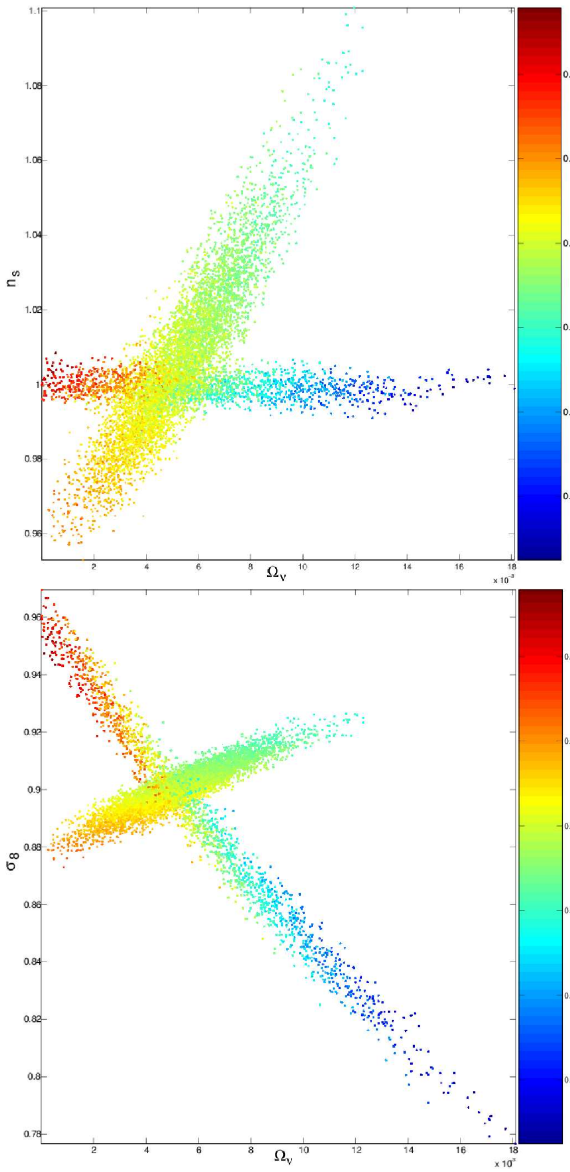

Fig. 2. We illustrate the main degeneracies present in LSS(SKA) and CMB(Planck) data, and how the combination of both approaches can greatly improve the errors on Ω_ν. We plot MCMC samples for both types of experiment with different parameters on the x and y axes. The top panels illustrates that the CMB measures n_s accurately, whereas there is a degeneracy between Ω_ν and n_s for LSS data. The bottom panel shows that there is an anti-correlation between the parameters σ_8 and Ω_ν for CMB data, whereas there is a positive correlation in the case of LSS data: in the case of the CMB, the Ω_ν - σ_8 degeneracy reflects the growth of structure from redshift $z \sim 1000$ to present epochs; for LSS, the Ω_ν - σ_8 degeneracy is due to the shape of the power spectrum.

very well independently of most other parameters. Hence a combination of these two types of dataset improve the error bars on the neutrino mass by a factor ~ 5, assuming $m_\nu \sim 0.25$ eV (Abdalla & Rawlings 2007).

Another degeneracy that plagues estimates of the neutrino mass via cosmological methods is the value of the amplitude of the initial fluctuations, parametrised by σ_8. We plot in Fig. 2 the σ_8 - Ω_ν degeneracy for CMB and LSS surveys. Whereas for CMB datasets, the growth of structure is responsible for the degeneracy between Ω_ν and σ_8, for a LSS survey it the shape of the power spectrum which is the main cause of degeneracy. By combining the two different types of datasets, a huge improvement can be achieved.

However a poor knowledge of the bias parameters, the ways in which HI-emitting galaxies trace dark matter as a function of redshift, would not allow us to combine these data sets in a consistent way, and the improvement would be smaller. We have assumed that the bias of high-redshift galaxies can be measured, and hence marginalised, via the redshift-space distortions induced by dark matter. Current spectroscopic surveys with much smaller samples are able to measure the bias to the few-percent level needed (Verde et al. 2006) so we argue that the assumptions we made here are not over-optimistic. We point out that the use of such techniques are a critical advantage of an 'all-sky' *spectroscopic* redshift survey over surveys employing photometric redshifts. We also point out that it is certainly possible that the SKA data at high redshift will allow us to probe smaller linear scales than that encoded in the $k_{\rm max}$ adopted here, particularly as methods for probing further into the non-linear regime of galaxy clustering are developed.

2.2.3. Directly probing neutrino hierarchies if $\sum m_i \sim 0.25$ eV

In our analysis of mock future datasets we have assumed that the number of massive neutrinos N_ν is constant. If we assume the standard scenario for neutrinos (see Fig. 1 of Abdalla & Rawlings 2007) this can arise when m_ν lies near the top of the currently allowed range, in which case the masses are quasi-degenerate ($N_\nu \approx 3$), or near the bottom when the mass hierarchy is either normal ($N_\nu \approx 1$) or inverted ($N_\nu \approx 2$).

Both LSS (SKA) and CMB (Planck) datasets do not on their own constrain N_ν significantly. However, when we combine both experiments we obtain a measurement of $N_\nu = 2 \pm 0.5$ (1σ) and would reject the value $N_\nu = 1$ at around the 3σ level. This combination of Ω_ν and N_ν would then yield invaluable information on the neutrino sector. We plot in Fig. 3 the constraints on both of these parameters from a combination of LSS(SKA) and CMB(Planck) experiments.

2.3. Indirectly probing neutrino hierarchies if $m_\nu \sim 0.05$ eV

The problem is significantly harder if the absolute neutrino mass scale turns out to be close to the lowest value ($m_\nu \sim 0.05$ eV) allowed by particle physics experiments. We have run MCMC chains that assumed a fiducial model $\{\Omega_b, \Omega_c, \Omega_\nu, h, n_s, \sigma_8, N_\nu\} = \{0.04, 0.259, 0.001, 0.72, 1.0, 0.9, 2\}$, in order to test whether future surveys will be able to determine hierarchies.

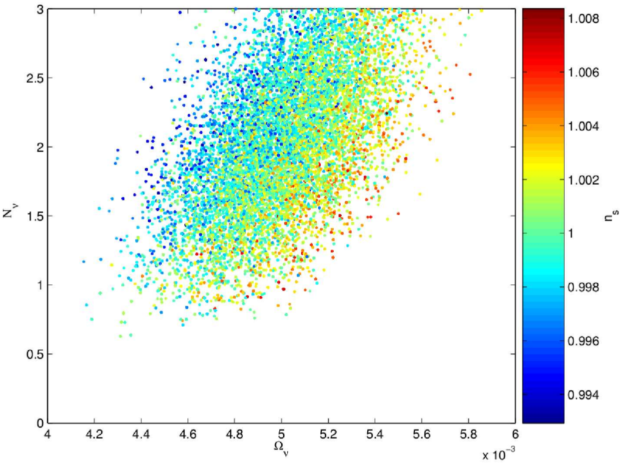

Fig. 3. Confidence limits on neutrino hierarchies with simulated LSS(SKA) and CMB(Planck) data. Illustration of the 2D distribution of MCMC samples in the $\{N_\nu,\Omega_\nu\}$ plane. The covariance around the fiducial {2,0.005} point is such that it is possible for us to discriminate between a normal ($N_\nu \approx 1$) and inverted ($N_\nu \approx 2$) neutrino hierarchy. For higher values for the fiducial Ω_ν discrimination is much clearer, and, as we can see from Fig. 4, for lower values of fiducial Ω_ν discrimination becomes impossible.

We find that for an SKA survey alone the relevant parameters will not be measured accurately enough to provide strong bounds on the neutrino mass. However, as detailed in Abdalla & Rawlings (2007), the bottom line of the joint LSS(SKA) and CMB(Planck) simulations is that they should together provide robust measurement of m_ν even if it lies towards the lowest end of the range currently allowed by particle physics experiments.

We plot in Fig. 4 the posterior from the MCMC samples for the parameters Ω_ν and N_ν. Even though Ω_ν is measured, N_ν is unconstrained. However, if $\sum m_i \lesssim 0.1$ eV then the hierarchy must be normal because, in an inverted hierarchy, the two most massive neutrinos would have similar mass, and the sum of neutrino masses would then exceed twice the minimum value of m_ν allowed by mass-splitting measurements from neutrino oscillations.

3. Conclusions

We have shown that future 'all sky' redshift surveys of LSS, e.g. with the SKA, will be able to detect a signal from neutrinos in the power spectrum of galaxies. We argue that, as datasets will be available from galaxies with different spatial clustering over different ranges of cosmic epoch, it will be possible to disentangle the effects of neutrinos from those of galaxy bias. We stress the importance of spectroscopic, rather than photometric, redshifts in measuring biases via velocity-space distortion.

The combination of LSS(SKA) and CMB(Planck Satellite) data will be particularly powerful in eliminating residual problems due to parameter degeneracies. We conclude that combining LSS(SKA) and CMB(Planck) will allow the measurement of the absolute mass scale of neutrinos in any standard scenario. Furthermore, if the sum of neutrino masses is below ~ 0.1 eV or above ~ 0.25 eV, it is likely that cosmological data will be able to determine the hierarchy.

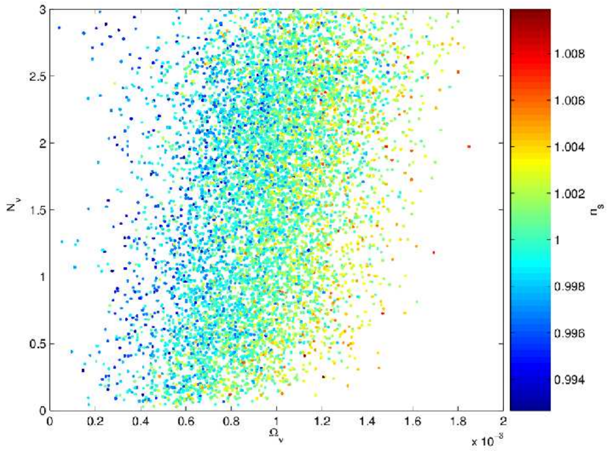

Fig. 4. Same as Fig. 3 in that it shows the 2D distribution of MCMC samples in the $\{N_\nu,\Omega_\nu\}$ plane, but different in that now the covariance is around the {2,0.001} fiducial point. It is no longer possible to probe the neutrino hierarchy directly. It is still possible to rule out the inverted hierarchy if the sum of the neutrino masses is below $\sum m_i \sim 0.1$ eV, i.e. twice the minimum value allowed by neutrino oscillation experiments.

Finally, we note that it may eventually be possible to measure exotic sterile particles that contribute to the cosmic background of relativistic particles, and hence influence $P(k)$, but which are not detectable by particle physics experiments.

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